On Optimality Criteria for Reverse Charging of Electric Vehicles

Sonja Stüdli, Wynita Griggs, Emanuele Crisostomi and Robert Shorten

Abstract—Ever increasing expectations regarding the penetration level of electric vehicles (EVs) are driving several areas of research concerning the problem of EV charging. One topic of interest treats EVs not only as controllable loads, but also as storage systems, which are able to reduce stresses on the grid during peak times by injecting power back. This is known as vehicle to grid. In this paper, we formulate the problem of returning electrical load to the grid as an optimisation whose goal is to minimise the effect of energy return on the environment.

I. INTRODUCTION

Awareness about greenhouse gases and air pollution in cities has increased in recent years [1], [2] and the shift to more environmentally friendly transportation systems is now a worldwide goal. Plug-in hybrids and full electric vehicles (EVs) are considered as “green” alternatives to the combustion engine, and the deployment of such vehicles is now widely encouraged [3]. This interest is driving several active areas of research including battery design, fast charging, grid-vehicle charge balancing, and distributed charging of fleets of electric vehicles. The main advantage of plug-in electric vehicles is that they allow us to control where and when emissions are released. Another purported advantage is that, due to the assumed high penetration levels [5], [9], [8], [10], [14], and [11], such vehicles can be used to store energy and deliver this energy back to the grid in times of need. This concept is usually referred to as vehicle to grid (V2G) and is considered as a key-point for implementing peak shaving and valley filling policies, and to reduce losses, see for example [4], [12], and [13]. While the ability of V2G to balance the demands of the grid, the availability of renewable energy, and the needs of commuters have been extensively investigated in the literature, little attention has been paid to the management of this energy flow; to date most work has focused on the support of the grid and neglects the consequences of the delivered power to the EVs. In particular, given a certain demand for energy from the grid, and an oversupply of available power from the fleet of electric vehicles, the manner in which energy is drawn from the fleet of electric vehicles may have a profound impact on the environment as well as on other individual commuters. In this paper we investigate such issues. Specific attention is paid to the various factors which have to be considered before drawing power from the EVs. These factors form a complex optimisation problem, where three key points need to be balanced: effect on the environment, disruption for the vehicle owner, and price.

II. NOMENCLATURE

The following terms are used throughout the paper:

- \( PHEV \) plug-in hybrid vehicle
- \( BEV \) full electric vehicle
- \( plant \) power plant
- \( i \) an index (denotes \( PHEV, \) \( BEV \) or \( plant \))
- \( E_i \) energy taken from \( i \) to supply the grid
- \( p \) pollution coefficient
- \( r_d \) desired driving distance
- \( r_a \) available driving distance in full electric mode
- \( d \) acceptable walking distance
- \( k \) adjustment factor for driver behaviour, route selection, weather forecast, extra individual power consumption
- \( l \) adjustment factor for energy losses due to conversion
- \( SOC \) state of charge of the vehicle battery
- \( \Delta E \) missing energy until battery is fully charged
- \( \bar{E} \) upper limit of energy deliverable by power plant
- \( E_{req} \) energy required by the grid
- \( c_1, c_2 \) energy price coefficients

III. MINIMISING IMPACT ON THE ENVIRONMENT

Consider the following categories of willing participants in an energy exchange programme with an electricity grid: full electric vehicles, plug-in hybrid vehicles and power stations. A goal is to supply the electricity grid with a necessary amount of energy, while choosing the energy in a way that minimises the impact on the environment caused by the energy transfer. For each participant, we will construct a utility function that describes impact on the environment in terms of emissions, where quantity of energy transferred to or from the participant is the utility function’s independent variable. These utility functions are then used to formulate an optimization problem.

A. Utility Functions

The three categories of participants involved in the energy exchange programme with the grid are: plug-in hybrid vehicles, full electric vehicles, and electricity generators (or power...
plants). With regards to energy transfer to or from each of these “types” of participants, the effects on the environment depend on various factors. We selectively list some of these factors according to each category of participant, and then use them to derive our sample utility functions. Note that we do not claim that our list of factors is complete, nor do we claim that each utility function is the unique possibility; rather, we aim to illustrate the variety of factors involved and the potential complexity of the optimization problem.

**Plug-in Hybrid Vehicles**

The environmental footprint of a plug-in hybrid vehicle depends on several factors. First, if the desired driving distance is greater than the distance that the vehicle can drive in full electric mode, then the driver will most likely switch to the vehicle’s combustion engine when electric energy is depleted, and this will have an impact on the environment through the use of carbon based fuels. By taking electric energy from the vehicle, we reduce the range in fully electric mode, and thus make it more likely that pollutants will be produced. Note that the available range in full electric mode is a complicated matter and depends on: the state of charge of the battery pack; the basic power consumption per kilometre; individual driving behaviour; and usage of other electrical appliances, for example heating, entertainment systems, headlights, or GPS. The driven route also has a strong influence on the available full electric range, as power consumption varies according to driving speed, the length of the journey and the topology of the terrain. For instance, [16] shows how driving range can be maximised by thoughtful route selection. Weather is another important factor. For instance, if the air conditioning system is in use, it consumes a large amount of energy, which is then not available for driving.

Once the vehicle switches to the internal combustion engine, then the car produces air pollution, namely particulate matter, CO and other carbon-related pollutants, as well as greenhouse gases, while driving. This production is dependent on the type of the car, the mode of vehicle operation, and the average speed of the vehicle. An important effect arises in some situations due to route choices that may depend on the availability of electric power. For example, in some German cities, Environmental zones (“Umweltzonen”) were introduced in 2008 [15]. The idea is that cars, producing too much particulate matter and other pollutants, are not allowed to enter a city zone. By taking electric energy from the vehicle, restrictions such as these could decrease the mobility of the owner or force him or her to make longer or faster journeys with an associated increase in pollution production. Finally, it is worth noting that transfer of energy also generates losses. This lost electric power is neither used to propel vehicles nor to power the grid, and must be regenerated. This regeneration also has an environmental cost.

Given these considerations, we now construct a sample utility function describing emissions due to energy transfer to or from a plug-in hybrid, as follows. Let \( r_a \) (i.e. available driving range in full electric mode) be a piecewise linear function of injected energy \( E_{PHEV} \):

\[
r_a(E_{PHEV}) = k(SOC - lE_{PHEV}),
\]

where \( l > 1 \) if \( E_{PHEV} \geq 0 \) and \( l < 1 \) otherwise. (Nomenclature concerning the parameters was given in Section II.) Suppose that \( p > 0 \) if \( r_d > r_a \) and \( p = 0 \) otherwise. Note that the factor \( p \) can be used to model either the air pollution, the \( CO_2 \) emissions, or a weighted combination of both. A sample utility function is then

\[
f_{PHEV}(E_{PHEV}) = p(r_d - k(SOC - lE_{PHEV})). \tag{1}
\]

The typical shape of our utility function (i.e: its convex and piecewise linear nature) is depicted in Figure 1.

**Full Electric Vehicles**

Some of the issues which arise for the plug-in hybrid car also concern the full electric vehicle in a similar way. For example, the expected demanded range has a direct influence on the feasibility of taking power from the owner, and on the environmental consequences of taking energy from the vehicle. As before, the available range itself is dependent on multiple factors:

- The state of charge.
- The basic power consumption per kilometre.
- The driven route.
- The individual driving behaviour.
- The weather condition.
- The usage of other electric appliances.

The consequence of taking energy from the EV owner will sometimes lead to behavioral change as in the worst case the owner has not enough energy to complete the desired route.
Therefore, the owner needs to use alternative transportation modes, which may cause more inconvenience for him/her and/or pollution. The alternatives, and the effect, for the environment are highly dependent on the owner and the region where he/she is at the moment. The basic alternatives and their influence are summarised below.

- **Recharging:** The owner can recharge the EV either on the journey, or keep it connected at home for an additional period. Recharging causes a delay, therefore it is only an option if the owner has not a fixed schedule. Additionally, the emissions, during the extra charging period, depend only on the generation side, and the extra power consumption may cause even worse environmental issues for the grid.

- **Second car:** the owner may have a second car available as a replacement. In this situation the additional pollution depends on whether it is a full electric vehicle, a plug-in hybrid, or a conventional combustion engine. It also depends on the specifics of the second car, for example the $CO_2$ emissions per $km$ for the combustion engine or the state of charge for an EV.

- **Public transport:** the feasibility of public transport as an alternative depends highly on the local availability, the costs and the estimated pollution by the public transport system. For example a highly developed and environmentally friendly system could even increase the environmental benefits, while keeping the inconvenience for the owner small.

- **Other measures:** if the owner has none of the above possibilities the inconvenience for the owner is extremely high and he/she will have to organise other transportation (e.g. taxi) or change his/her plans (take an hour off from work, reschedule a meeting).

Using the factors above an example utility function is constructed. The factors are computed similar to the plug-in hybrid version. In other words, $r_a(E_{BEV}) = k(SOC - l E_{BEV})$, where $l > 1$ if $E_{BEV} \geq 0$ and $l < 1$ otherwise. Further, it is assumed that the owner has only one alternative, so in the case the energy is not enough to drive the full way the owner takes this alternative for the rest of the way. We assume that a distance $d$ is the maximum walking distance, if the missing range is smaller than $d$, it does not cause any pollution. Otherwise it causes pollution per missing $km$.

A sample utility function for the pollution is then

$$ f_{BEV}(E_{BEV}) = p(r_d - d - k(SOC - l E_{BEV})) $$

(2)

if $E_{BEV} > \frac{kSOC - r_d + d}{l} \> 0$ and 0 otherwise. Some sample utility functions are depicted in Figure 2.

**Power Plants**

The aspects of the problem considered thus far refer to issues on the consumer side. Since a supplier may have the option to generate rather than recover power, or to purchase from a another generation company, the ability to obtain electricity through generation must also form part of the optimization problem as this may in some situations be preferable to recovering energy from the vehicles. Generators differ from vehicles in that they are designed to deliver power only. Nevertheless, we can still model the environmental impact of a generator in terms of a utility function. Note that we only consider generators which are able to regulate their power output. We now consider the construction of utility functions to describe power plants.

**Pollution:** The first term in the utility function accounts for air pollution and $CO_2$ emissions which are caused by the power plant adapting its power output. Here it is also important to consider pollution which is caused by changing the output.

**Waste:** the generation of energy produces some amount of waste, for which the disposal has to be taken into account (the cost and the negative environmental effects).

**Raw materials:** as most generators depend on raw materials, the pollution, the effects on the environment, and the cost of its production and transportation has to be taken into account.

To construct a utility function we assume a linear relationship between pollution, the energy delivered $E_{plant}$ and its production $p E_{plant}$. This results in the utility function

$$ f_{plant}(E_{plant}) = p E_{plant} $$

(3)

Note that in the factor $p$ the resource and waste are taken into account.

**B. Optimisation Problem**

In the following three examples we illustrate the type of optimisation issues that arise in the reverse charging problem.
We assume in all examples that the goal is to feed 18 kWh to the grid.

**Example 1: Equal Contribution:** In this first example, we consider a naive approach to the problem. The effect on the environment is neglected and the required energy by the grid is delivered equally by all contributing vehicles. Suppose that three vehicles are participating in the energy exchange with the rest of the grid. One of the vehicles is a plug-in hybrid and the other two are full electric vehicles. The various values used to determine the pollution cost are summarised in Table I under the entries BEV1, BEV2, and PHEV1 for the two electric vehicles and the plug-in hybrid, respectively. The resulting environmental costs are summarised in Table II. The total cost to the environment is 127.36 g, and this cost is entirely due to BEV1 and PHEV1.

**Example 2: Pollution Minimisation:** We now repeat the above example within an optimization framework. We now wish to minimize the environmental cost of the reverse charging problem. Thus, the goal is to minimize the total pollution, and the sum of the utility functions becomes our objective function. The optimization problem thus becomes:

$$\min_{s.t. \sum E_{\text{BEV}} + \sum E_{\text{PHEV}} = E_{\text{req}}} \sum f_{\text{PHEV}} + \sum f_{\text{BEV}},$$

where $E_{\text{req}}$ is the required total energy (by the grid) for the next time period. Additionally the optimization variables are subject to additional SOC constraints:

$$-\Delta E_{\text{BEV}}^{(i)} \leq E_{\text{BEV}}^{(i)} \leq SOC_{\text{BEV}}$$

$$-\Delta E_{\text{PHEV}}^{(i)} \leq E_{\text{PHEV}}^{(i)} \leq SOC_{\text{PHEV}}.$$

Suppose now two full electric vehicles and one plug-in hybrid are prepared to deliver energy to the grid, as in Example 1. The values used for the utility function are summarised in Table I and are identical to the ones from Example 1. The minimisation problem is solved using the MATLAB optimization toolbox and the results are summarised in Table III. The total pollution is reduced to 75.81 g, which is a reduction of more than 40% compared to the naive solution. Most of the pollution is caused by BEV 1, while PHEV 1 makes a very small contribution to the required energy. Thus we see that a judicious choice of how the energy is recovered by the grid makes a huge difference to the environment.

**Example 3: Pollution Minimisation including Power Plants:** Finally, we now consider the effect of allowing the power management company to switch on new generating capacity. As before, the sum of the individual utility functions, including the environmental costs caused by power plants, is our objective function to be minimised. The problem now is how to draw energy for the next time-step of the different parties, which minimises the impact on the environment. The optimisation problem thus is

$$\min_{s.t. \sum E_{\text{PHEV}} + \sum E_{\text{BEV}} + \sum E_{\text{plant}} = E_{\text{req}}} \sum f_{\text{PHEV}} + \sum f_{\text{BEV}} + \sum f_{\text{plant}},$$

where $E_{\text{req}}$ is the needed total energy for the next time period. Additionally the optimisation variables are bounded by

$$-\Delta E_{\text{BEV}}^{(i)} \leq E_{\text{BEV}}^{(i)} \leq SOC_{\text{BEV}}$$

$$-\Delta E_{\text{PHEV}}^{(i)} \leq E_{\text{PHEV}}^{(i)} \leq SOC_{\text{PHEV}}$$

$$0 \leq E_{\text{plant}}^{(i)} \leq E_{\text{plant}}.$$

where $\Delta E_x$ is the required energy until the battery is fully charged and $E_{\text{plant}}$ is the maximal energy that can be delivered from plant $i$. Suppose that three vehicles and one small power plant are prepared to participate in an energy exchange with the grid: two of the vehicles are full electrics and one is a plug-in hybrid. The various parameter values for the vehicles and the power plant are summarised in Table I using the same values for the vehicles as in the previous two examples. We solve the emissions minimisation problem using MATLAB. The results are provided in Table IV. According to the optimisation solution, energy should be supplied to the grid from full electric vehicle 2 (BEV 2) and from the power plant (plant 1), while some energy should be delivered to full electric

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>PARAMETER VALUES FOR PARTICIPATING VEHICLES AND POWER PLANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$ [g/km]</td>
<td>BEV 1</td>
</tr>
<tr>
<td>3.8</td>
<td>5.6</td>
</tr>
<tr>
<td>$p$ [g/kWh]</td>
<td>n/a</td>
</tr>
<tr>
<td>$r_d$ [km]</td>
<td>19</td>
</tr>
<tr>
<td>$d$ [km]</td>
<td>0.4</td>
</tr>
<tr>
<td>$k$ [km/kWh]</td>
<td>2</td>
</tr>
<tr>
<td>$l$ ($E_i \geq 0$)</td>
<td>1.05</td>
</tr>
<tr>
<td>$l$ ($E_i &lt; 0$)</td>
<td>0.95</td>
</tr>
<tr>
<td>$SOC$ [kWh]</td>
<td>6</td>
</tr>
<tr>
<td>$\bar{E}$ [kWh]</td>
<td>14</td>
</tr>
<tr>
<td>$c_1$ [$S/kWh^2$]</td>
<td>0.5</td>
</tr>
<tr>
<td>$c_3$ [$S/kWh$]</td>
<td>n/a</td>
</tr>
<tr>
<td>$c_9$ [$S/g$]</td>
<td>0.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>EQUAL CONTRIBUTION: ENERGY CONTRIBUTION AND RESULTING ENVIRONMENTAL COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_i$ [kWh]</td>
<td>BEV 1</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>$f_i$ [g]</td>
<td>72.96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>POLLUTION MINIMISATION: ENERGY CONTRIBUTION AND RESULTING ENVIRONMENTAL COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_i$ [kWh]</td>
<td>BEV 1</td>
</tr>
<tr>
<td>5.7648</td>
<td>12.1487</td>
</tr>
<tr>
<td>$f_i$ [g]</td>
<td>71.0831</td>
</tr>
</tbody>
</table>
vehicle 1 (BEV 1) and the plug-in hybrid (PHEV 1) for their trips. The total pollution emitted due to the energy exchange is 54.1823 g, which is a reduction of over 55% compared with the equal contribution example.

**Comment:** A very interesting feature of the previous example is that it is preferable to generate new energy than to take all energy from the plug-in fleet.

**Comment (utility fairness):** In a dynamic market situation where users sell energy back to the grid, the above optimization may be very unsatisfactory for individual users and cause much disruption to certain customer types. For example, a utility company would frequently drain energy from low polluting cars, resulting in these vehicle owners frequently having to make alternative arrangements for unexpected trips. Under this scheme, the batteries of low polluting cars also undergo more frequent charge cycles, degrading battery life more quickly. Meanwhile, higher polluting vehicles are not penalised at all. Of course, such users probably have a financial benefit. Nevertheless, one alternative method to achieve fairness in the network is to use the utility functions to dictate how much energy each user gives back to the network: this is known as utility fairness [17]. Figure 3 illustrates this idea. Here we ensure that the environmental cost to each user is the same. This is now no longer a minimization problem, but rather an equalisation problem and can be solved either in a centralized manner, or in a decentralised manner using the implicit consensus techniques in [17], [18].

![Utility fairness](image)

**IV. ALTERNATIVE OPTIMALITY CRITERIA**

The optimisation in the previous section reflects the desire to minimise the generation of additional greenhouse gases as a result of taking energy from the fleet of battery powered vehicles. Many other important optimality criteria can be formulated, depending, perhaps, on regulatory requirements, and on issues that are of interest to the grid operator. Such considerations might reflect the generation of localised pollutants, as well as the financial cost to the grid operator. Other factors that might be included in the optimization are the increased pressures on public transport, and increased congestion that may accrue as a result of reduced EV activity in certain city areas (i.e. more traffic would be forced away from green areas). In practice, any realistic optimality criterion would reflect all of these issues (including the issue of fairness) in a combined manner. A basic optimization problem would then be to balance a tradeoff between generating emissions and the cost to the utility [17]. To illustrate one such tradeoff we conclude our paper by illustrating a simple alternative optimisation arising from the desire to minimise financial cost to the grid operator. We consider again the following three categories of participants involved in the energy exchange with the grid: plug-in hybrid vehicles, full electric vehicles, and a power plant. Before proceeding it is worth noting at this stage that a utility company can always set a total fixed amount that it is willing to pay for energy, and that the emissions minimisation problem of the previous section can be solved with this additional price constraint.

As an incentive to a vehicle owner to participate in the energy exchange programme, the price at which energy is sold back to the utility company should be at least as high as the expected costs to the owner: to recharge the battery; to purchase fuel to replace the energy drawn from the battery; or to take alternative transport (e.g. public transport or taxis), etc. As an incentive to the power plant to participate in the energy exchange programme, it needs to re-cooperate its costs for extra energy production, e.g. the purchase of more coal and equipment maintenance. In addition, suppose that both vehicle owners and plants are taxed by government per gram of emissions caused by sending energy to the grid. The participants pass this tax onto the utility company in addition to their previously mentioned expenses. Hence, let us derive sample utility functions that describe financial cost to the utility company for each vehicle or plant participant.

Plug-in hybrid vehicle owners charge the utility company for energy as follows:

$$g_{PHEV}(E_{PHEV}) = c_1 E_{PHEV}^2 + c_2 f_{PHEV}(E_{PHEV}),$$

where $c_1$ and $c_2$ are energy price coefficients. Specifically, $c_1$ covers vehicle re-fuelling or public transport costs and the general inconvenience experienced by the vehicle owner in selling energy back to the grid. With respect to inconvenience, particularly note the rate at which the first part of the utility function monotonically increases, to reflect the notion that drawing only a little energy from a vehicle is not as likely to disrupt a vehicle owner’s travel plans, whereas taking a lot of energy is likely to have a larger impact on the vehicle owner. Hence, the higher the price that the vehicle owner demands for the more energy drawn is compensation for the vehicle owner.
utility minimization or equalisation problem. In our paper, we focus on the environmental effects of drawing energy from a fleet of battery powered vehicles. Surprisingly, we show that in some situations, it is better to increase the power output of a generation plant rather than draw the power from the vehicles. Future work will investigate the multi-variate optimality criteria alluded to in the paper.

The coefficient $c_2$ describes the government tariff imposed on the vehicle owner with respect to emissions caused by transferring energy, which the vehicle owner passes on to the utility company. Similarly, suppose that utility functions for the full electric vehicles and power plant are as follows:

$$g_{BEV}(E_{BEV}) = c_1 E_{BEV}^2 + c_2 f_{BEV}(E_{BEV}),$$

and

$$g_{plant}(E_{plant}) = c_1 E_{plant}^2 + c_2 f_{plant}(E_{plant}).$$

Suppose that the government tariff on emissions due to power plants sending energy to the grid is higher than the tariff on vehicle owners.

The optimisation problem is then

$$\text{MINIMISING ENERGY PRICE}$$

$$\text{TOTAL COST} = \sum g_{BEV} + \sum g_{BEV} + \sum g_{plant}$$

s.t. \[ \Delta E_{PHEV} \leq E_{PHEV}^{(i)} \leq SOC_{PHEV} \]

\[ \Delta E_{BEV} \leq E_{BEV}^{(i)} \leq SOC_{BEV} \]

\[ 0 \leq E_{plant}^{(i)} \leq \bar{E} \]

$$\sum E_{PHEV} + \sum E_{BEV} + \sum E_{plant} = E_{req}$$

We solve an example using the General Algebraic Modeling System (GAMS) software, Distribution 23.7.3, as follows.

**Example 4:** Consider three vehicles and a power plant as willing participates in an energy exchange programme with the grid: two vehicles are full electrics and one is a plug-in hybrid. Values for the energy price coefficients $c_1$, $c_2$ and all of the other parameters are provided in Table I. Suppose that the grid again requires 18 kWh of energy. The results of the optimisation are summarised in Table V. Two locally optimal solutions are found. The minimum cost to the grid company for the energy is given as $\$54,418$. This is achieved when 2,602 kWh of energy is taken from full electric vehicle 1 (BEV 1), 3.4 kWh of energy is taken from full electric vehicle 2 (BEV 2), 1,075 kWh of energy is taken from plug-in hybrid 1 (PHEV 1) and 10,923 kWh of energy is taken from power plant 1. Note that, with these energy values, 118.95 g of emissions total are produced, more emissions than what were produced in Example 3.

**TABLE V**

<table>
<thead>
<tr>
<th></th>
<th>BEV 1</th>
<th>BEV 2</th>
<th>PHEV 1</th>
<th>plant 1</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_i$ [kWh]</td>
<td>2.602</td>
<td>3.4</td>
<td>1.075</td>
<td>10,923</td>
<td>18</td>
</tr>
<tr>
<td>$g_i$ [S]</td>
<td>7.97</td>
<td>5.78</td>
<td>3.53</td>
<td>37.138</td>
<td>54.418</td>
</tr>
</tbody>
</table>

**V. CONCLUDING REMARKS**

In this paper we give a new perspective on the V2G concept. Given a certain level of demand from the grid, and a fleet of EVs and other participants, there are many ways in which this energy can be drawn from participants so as to satisfy the demands of the grid. By introducing a notion of utility, the manner in which the energy is drawn from each participant can be uniquely defined by solving a

**REFERENCES**


